Key

Math 4 Name ______________________

1-3 Parametric Equations Date ________

Learning Goals:

- *I can write and graph parametric equations.*
- *I can solve applications involving parametric equations.*

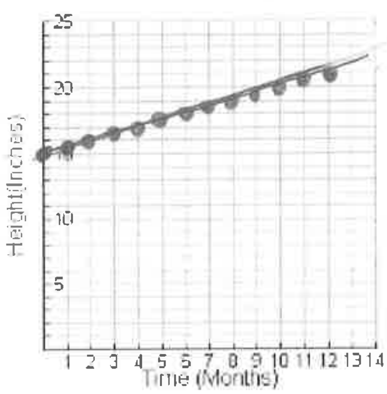
Parametric equations for a curve are equations in which the x and y coordinates are both expressed in terms of a single variable, called the parameter, t. In "real-life" applications, t often represents time.

I. Consider the following scenario:
A puppy weighs 8 pounds and is 15 inches long at birth. For its first year of life, each month the puppy grows .5 inches and gains 3 pounds. Complete the following table:

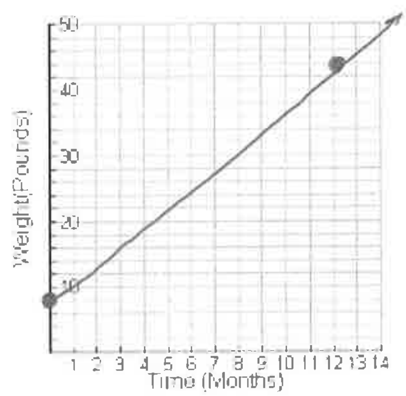
Time (months)	0	1	2	3	4	5	6	7	8	9	10	11	12	Rules:
Height (in.)	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	$X(t) = 15 + 0.5t$
Weight (lbs.)	8	11	14	17	20	23	26	29	32	35	38	41	44	$y(t) = 8 + 3t$

A. Sketch the following graphs:

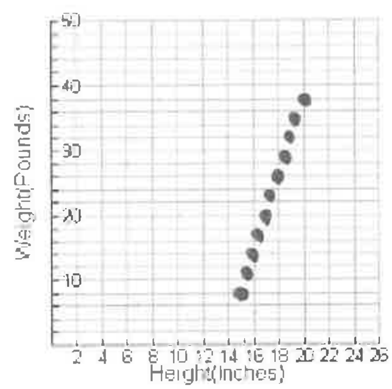
Time vs. Height



Time vs. Weight



Height vs. Weight



Parametric equations for this scenario: $\begin{cases} x(t) = 15 + 0.5t \\ y(t) = 8 + 3t \end{cases}$

B. See if you can reproduce your *Height vs. Weight* graph on your calculator. Follow these tips . . . .

***You will receive a ½-sheet of paper showing you the finer points of graphing parametric equations on the TI-*nspire*. **Please read the directions carefully.**

Some things to consider . . . .

Menu-3-3

Adjust your *window*. Use graph #3's axes as a guide.
What will you enter for t minimum and maximum values? $0 \le t \le 12$
What about your *tstep*? $\longrightarrow$ tstep = 1
Not sure?
Experiment . . . .

C. Both parametric equations can be combined into one rectangular equation (in terms of x & y only.) This is called **eliminating the parameter, *t*.** To do this, start by solving the x-equation for t. Substitute the expression for t into the y-equation & simplify. *Show your work below.* Test your new rectangular equation by graphing it.

$X = 15 + 0.5t$

$(x - 15 = 0.5t) \cdot 2$

$2x - 30 = t$

$Y = 8 + 3(2x - 30)$

$Y = 8 + 6x - 90$

$\boxed{Y = 6x - 82}$

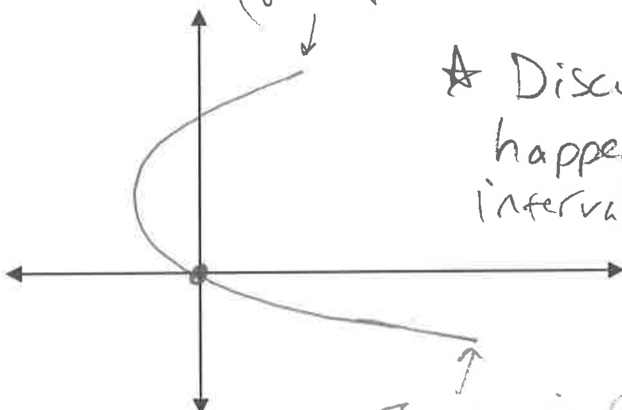
II. Applications:

A. Parametric equations enable us to graph a curve that may double back on itself or cross itself. Such a curve cannot be described by a function $y = f(x)$. The following examples are curves that are not functions in the rectangular system.

1. Graph the curved defined by the following set of equations over the interval $-2 \le t \le 5$ in your calculator. Make a sketch of the graph. t-Step = 1 (terminal point... calculus)

$$\begin{cases} x(t) = t^2 - 4t \\ y(t) = 3t \end{cases}$$

Window:
Xmin: -5
Xmax: 20
Ymin: -8
Ymax: 20



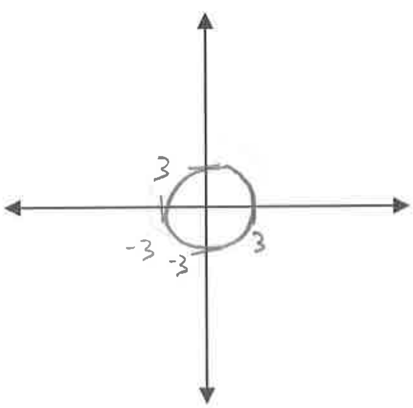
☆ Discuss what happens when changing intervals & tstep ☆

Initial point of graph (initial point - calculus)

2. Make sure your calculator is in radians. Graph the curve defined by the set of equations to the right over the interval $0 \le t \le 2\pi$. Use a *tstep* of π/6. Make a sketch of the graph.

$$\begin{cases} x(t) = 3\cos t \\ y(t) = 3\sin t \end{cases}$$

X: -5 to 5
Y: -5 to 5



Try Zooming in/out to get a good window

What equations would yield the *unit circle*? What *tstep* would you use?

X(t) = cos t

Y(t) = sin t

3. Here is another interesting example. Change the *tmax* to 12 π then graph. Adjust the window so you can see the full curve.

$$\begin{cases} x(t) = 11\cos t - 6\cos\left(\dfrac{11}{6}t\right) \\ y(t) = 11\sin t - 6\sin\left(\dfrac{11}{6}t\right) \end{cases}$$

Crazy awesome graph!
Can zoom out if necessary

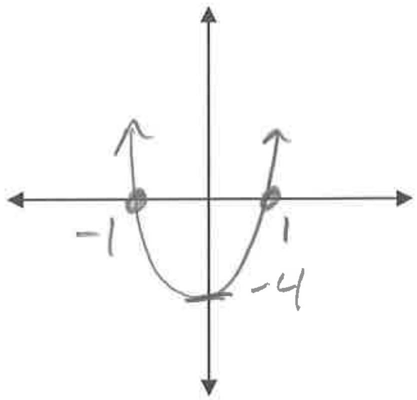
B. Eliminating the Parameter

Sketch the curve represented by the parametric equations $\begin{cases} x = \dfrac{t}{2} \\ y = t^2 - 4 \end{cases}$

Copy the sketch below.

t interval: $-5 \le t \le 5$

t step: 1



It looks very similar to another graph in rectangular form. To find that equation, do the following.

i. In one of the equations, solve for *t*. You decide which one looks easier.
ii. Substitute that value for *t* into the other equation and solve.

$X = \frac{t}{2} \rightarrow 2x = t$

$\downarrow$

$y = (2x)^2 - 4$

$= 4x^2 - 4 = 4(x^2-1) = 4(x+1)(x-1)$

Practice: Sketch the curve represented by the equations below by eliminating the parameter and adjusting the domain of the resulting rectangular equation.

$\begin{cases} x = \dfrac{1}{\sqrt{t+1}} \\ y = \dfrac{t}{t+1} \end{cases} \rightarrow x^2 = \frac{1}{t+1}$

$x^2(t+1) = 1$

$t+1 = \frac{1}{x^2}$

$t = \frac{1}{x^2} - 1$

$y = \dfrac{\frac{1}{x^2} - 1}{\left(\frac{1}{x^2} - 1\right) + 1}$

$y = \dfrac{\frac{1}{x^2} - 1}{\frac{1}{x^2}}$

$y = \left(\frac{1}{x^2} - 1\right) \cdot \frac{x^2}{1}$

$y = 1 - x^2 = -(-1 + x^2) = -(x^2-1)$

$= \boxed{-(x+1)(x-1)}$

Math 4 Name________________

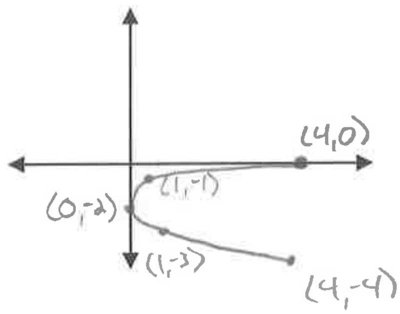
1-3 Parametric Equations Practice Date________

1. Make a table of values for the curve defined by the following set of equations over the interval $-2 \le t \le 2$.

$$\begin{cases} x(t) = t^2 \\ y(t) = t - 2 \end{cases}$$

Sketch a graph of the curve.

t	x	y
-2	4	-4
-1	1	-3
0	0	-2
1	1	-1
2	4	0



No arrows due to the interval (NOT continuous)

Write the parametric equations below as a single equation in *x* and *y* by eliminating the parameter, *t*. Check your result by showing that its graph and the graph of the parametric equations are the same.

2. $\begin{cases} x = 3 - 2t \\ y = 2 + 3t \end{cases}$

$X - 3 = -2t$

$\frac{X-3}{-2} = t$

$\frac{-x+3}{2} = t$

$y = 2 + 3\left(\frac{-x+3}{2}\right)$

$Y = 2 + \frac{-3x+9}{2}$

$Y = 2 + \frac{-3x}{2} + 4.5$

$\boxed{Y = 6.5 - 1.5x}$ ✓

3. $\begin{cases} x = t + 2 \\ y = t^2 - 3t \end{cases}$

$t = x - 2$

$Y = (x-2)^2 - 3(x-2)$

$Y = x^2 - 4x + 4 - 3x + 6$

$\boxed{Y = x^2 - 7x + 10 = (x-5)(x-2)}$

✓